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The calculation of long
distance transmission lines

Electrical Engineering

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**THE CALCULATION OF LONG DISTANCE
TRANSMISSION LINES**

BY

NELSON ROGERS BARR

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE, 1910 *m*

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

NELSON ROGERS BARR

ENTITLED THE CALCULATION OF LONG DISTANCE TRANSMISSION LINES

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

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LONG DISTANCE TRANSMISSION CALCULATIONS.

OBJECT.

The object of this thesis is to furnish such constants that the process of calculation of a transmission line is a few steps. The constants given apply for any voltage, and for the spacing and wires most common in transmission work. It also shows at what distance transmitted the method of using one half of the charging current, concentrated at the end of the line fails to apply, and what difference it makes to use distributed capacity and inductance.

REFERENCES.

The constants and their derivation were given by Dr. Berg, they being made adaptable to numerical calculation by him from Dr. Charles P. Steinmetz investigations given in his book called Alternating Current Phenomena page 155. Call number 537.83. Literature on line calculation, and some of the difficulties met with on transmission lines, is found in the following articles. Proceedings of the American Institute of Electrical Engineers, October, 1898, page 531. November 1909, page 1460 written by Mr. Charles F. Scott. Proceedings of the American Institute of Electrical Engineers, June 1909, page 509 and 483, written by Mr. Percy H. Thomas. Electrician London, February 12, 1909, page 686, written by Mr. J. O. Howe.

CONCLUSIONS.

As can be seen from the sample transmission line and the values for others, the distance transmitted before a variation in the two methods appears is under 50 miles or in other words the method of assuming $1/2$ of the charging current as concentrated at the end of the line is not an accurate method but for short lines.

EXPLANATION OF CURVES AND CALCULATIONS.

The curves are given for A. B. C. and D. for both spacings so any distance other than the ones given in the tables can be calculated.

The curve for A. (one spacing) is the same for all wires so only one is given; the curve for D. is not drawn for a distance shorter than 100 miles, as the scale necessary to use will not respond to the slight change in values below 100 miles, and distances under 100 miles are not often used.

A sample calculation from start to finish is given showing how the constants were gotten and how to use them as well as a comparison of the two methods spoken of above.

Following is a table giving the values of voltage and current for 00 wire both .9 lagging and leading power factor; also the per cent variation between the voltages calculated by the two methods.

Table of values of voltage & current calculated for 50 miles;
60,000 volts; 20,000 KW; 3 phase; 60 cycles; 10 ft. spacing;
.9 lag + .9 lead P. F

	Miles	I lag	I lead	E lag	E lead	%	%	Lag Lead % Diff.
(a)	50	213	215	37520	36370	8.43	4.82	14.27 -.48
(b)	50	212	216	42800	36100	23.7	4.34	
(a)	200	94.8	143	79700	68200	14.86	-1.73	94 10.22
(b)	200	97.0	126.4	80200	77700	15.8	11.95	
(a)	600	250	308	75700	68000	-24.2	-42.1	.9 -14.5
(b)	600	247	141.6	85000	83400	-27.1	-27.6	

(a) Represents the values gotten using distributed inductance and capacity.

b Represents the values gotten $1/2$ the charging current concentrated at the end of the line.

The percents are gotten on the basis of receive voltage.

The sign of the % difference indicates whether the value of (b) is less than (a) or not.

(a) and (b) for 200 and 600 miles has voltages 200 miles = 120,000
600 miles 200,000; KW the same through out.

Sample Calculation.

Size of wire : No 1 Voltage = 100 000 Power 20 000 K.W.

Three phase. Length of line 25 miles.

Distance between wires 10 feet.

r for 1 mile	.644
r for 25 miles	16.1
x for 1 mile	.85
x for 25 miles	21.25
r^2	259.2
x^2	451.56
$Z = \sqrt{r^2 + x^2}$	264.06
I_c : full charging current	7.275
$b = I_c / 58000$	.0001254
$(Z - x)$	5356
$.5b(Z - x)$	.0003364
$\alpha = \sqrt{.5b(Z - x)}$	.01831
$Z + x$	45.856
$.5b(Z + x)$	.003
$\beta = \sqrt{.5b(Z + x)}$	.05488
β_0	3° 8'
$r_0 = \beta / b$	437.7
$x_0 = \alpha / b$	146.0
$Z_0 = \sqrt{r_0^2 + x_0^2}$	461.28

$.434 \alpha$	.007945
$K = \text{corresponding number to } .007945$	1.018
$\cos \beta_0$	.9985
$\sin \beta_0$	.05466
$m_1 = K \cos \beta_0$	1.016
$m_2 = K \sin \beta_0$	.0556
$n_1 = \cos \beta_0 / K$	.9808
$n_2 = \sin \beta_0 / K$	.0536
$A = .5(m_1 + n_1)$	.9984
$B = .5(m_1 - n_1)$	.0176
$C = .5(m_2 + n_2)$	.0546
$D = .5(m_2 - n_2)$	.001

The following is a key to the above calculations and the constants calculated from these values.

$r =$ Resistance of the wire.

$x =$ Reactance of the wire.

$E =$ Voltage between wires divided by $\sqrt{3}$ for three phase work

$b =$ Capacity susceptance $= I_0 / E$

$\alpha = \sqrt{.5b(2-x)}$ and is expressed in radians

$\beta = \sqrt{.5b(2+x)}$ " " " " "

$\beta_0 = \beta \times 57.3$ and is expressed in degrees as 1 radian $= 57.3^\circ$

i = Energy current per phase at receiving end.

$I = i / \cos \phi$ = Total current per phase of load.

$\cos \phi$ = Power factor at receiving end.

E = Electromotive force per phase at receiving end (wire to neutral)

E_0 = " " " " " generator " " " "

I_0 = Total current per phase at generator.

$$a = I r_0$$

$$b = I x_0$$

$c = E \cos \phi$ where E = voltage between wires one square inch of three.

$$d = E \sin \phi$$

$$\tan u = u_2 / u_1$$

$$\tan E_0 = e_2 / e_1$$

$$\text{Angle } \Phi = \text{Angle } E_0 - I_0$$

$\cos \Phi$ = Power factor at generator.

$$\text{Angle } I_0 = \text{angle } u + \text{angle } z_0$$

$$\tan z_0 = x_0 / r_0$$

$$u = \sqrt{u_1^2 + u_2^2}$$

$$E_0 = \sqrt{e_1^2 + e_2^2}$$

$$I_0 = u / z_0$$

$$u_1 = Aa + Bc + Db - Cd$$

$$u_2 = Da + Cc - Ab + Bd$$

$$e_1 = Ba + Ac + Cb - Dd$$

$$e_2 = Ca + Dc - Bb - Ad$$

Length of wire?

Sample calculation making use the values (60 cycles) calculated.

Number of wire voltage =
40,000 power = 20,000 K.W. spacing of the wires = 10'.

Power factor

.9 lag

.9 lead

$$I = i / \cos \phi$$

214.00

214.00

$$a = I r_0$$

87000.00

87000.00

$$b = I x_0$$

20670.00

20670.00

$$c = E \cos \phi$$

31200.00

31200.00

$$d = E \sin \phi$$

15100.00

-15100.00

$$B a$$

2120.00

2120.00

$$A c$$

31000.00

31000.00

$$C b$$

248.00

248.00

$$\Sigma (B a, A c, C b)$$

33348.00

33348.00

$$D d$$

71.5

71.5

$$E_1$$

33296.00

33439.00

$$C a$$

1050.00

1050.00

$$D c$$

148.00

148.00

$$A d$$

15025.00

-15025.00

$$\Sigma (C a, D c, A d)$$

16192.00

-13883.00

$$- B b$$

503.00

503.00

$$e_2$$

15713.00

14884.00

$$e_1^2$$

11410000.00

11800000.00

$$e_2^2$$

24690000.00

20500000.00

$$E_o = e_1^2 + e_2^2$$

I_o

Using $1/2$ the charging current concentrated at the end of the line. Equation $I_o = \sqrt{I_1^2 + (I_c \pm i_c)^2}$. + for leading power factor, - for lagging.

X = Total reactance of line

r = Total resistance of line

i_c = $1/2$ charging current (wattless component)

i = Energy component of current

i_1 = Wattless component of current

E = Receiver voltage

i_r = Energy component $\times$ total resistance

$i_1 \times$ = Wattless component $\times$ total reactance

$-i_c \times$ = $1/2$ charging current $\times$ total reactance

$i \times$ = Energy component $\times$ Total reactance

$+i_c r$ = $1/2$ charging current $\times$ Total resistance.

$-i_r$

E_o

I (using i_c as 4.54 amperes)

37520.

34370.

213.

215.

41.0

41.0

20.15

20.15

4.54

4.54

192.5

192.5

93.7

93.7

34400.0

34400.0

3880.0

3880.0

3840.0

38480.0

42320.0

-14 3840.0

-170.0

34840.0

42150.0

-168 -170.0

7890.0

34470.0

80.0

14 7890.0

7970.0

i_r 1890.0

1890.0

$i_c \times$ 80.0

6080.0

9840.0

42800.0

34100.0

212.0

216.0

No. 1 WIRE. 10 FT. SPACING.
60 CYCLES.

Distance in Miles	25	50	100	200	400	600
r_0	437.7	437.7	437.7	437.7	437.7	437.7
χ_0	146.0	146.0	146.0	146.0	146.0	146.0
z_0	461.28	461.28	461.28	461.28	461.28	461.28
m_1	1.0160	1.0315	1.05017	1.048	0.8521	0.3864
m_2	0.0556	0.1154	0.2359	0.49018	1.027	1.502
n_1	0.9808	0.96	0.91	0.785	0.478	0.1606
n_2	0.0536	0.1075	0.202	0.366	0.578	0.624
$A = .5(m_1 + n_1)$	.99821	.99479	.98055	.91655	.67592	.3029
$B = .5(m_1 - n_1)$	.0176	.03525	.0701	.1315	.18705	.1129
$C = .5(m_2 + n_2)$	.0546	.1145	.21895	.42809	.8015	1.063
$D = .5(m_2 - n_2)$	.001	.00395	.01695	.06209	.0225	0.389

No. 0 WIRE. 10 FT. SPACING.
60 CYCLES.

Distance in Miles	25	50	100	200	400	600
r_0	421.	421.	421.	421.	421.	421.
x_0	120	120	120	120	120	120.
z_0	437.7	437.7	437.7	437.7	437.7	437.7
m_1	1.013	1.025	1.038	1.028	0.835	0.4066
m_2	.06473	.1098	.2246	.4688	.96	1.384
n_1	.9853	.964	.92	.8051	.515	.1952
n_2	.0638	.1032	.199	.367	.592	.665
$A = .5(m_1 + n_1)$	.9984	.99479	.98056	.91655	.67592	.3029
$B = .5(m_1 - n_1)$	.01385	.0305	.059	.11145	.160	.1052
$C = .5(m_2 + n_2)$	.06426	.1065	.2118	.4169	.776	1.0245
$D = .5(m_2 - n_2)$	.000468	.0033	.0128	.0509	.184	.3595

No.00 WIRE. 10 FT. SPACING.
60 CYCLES.

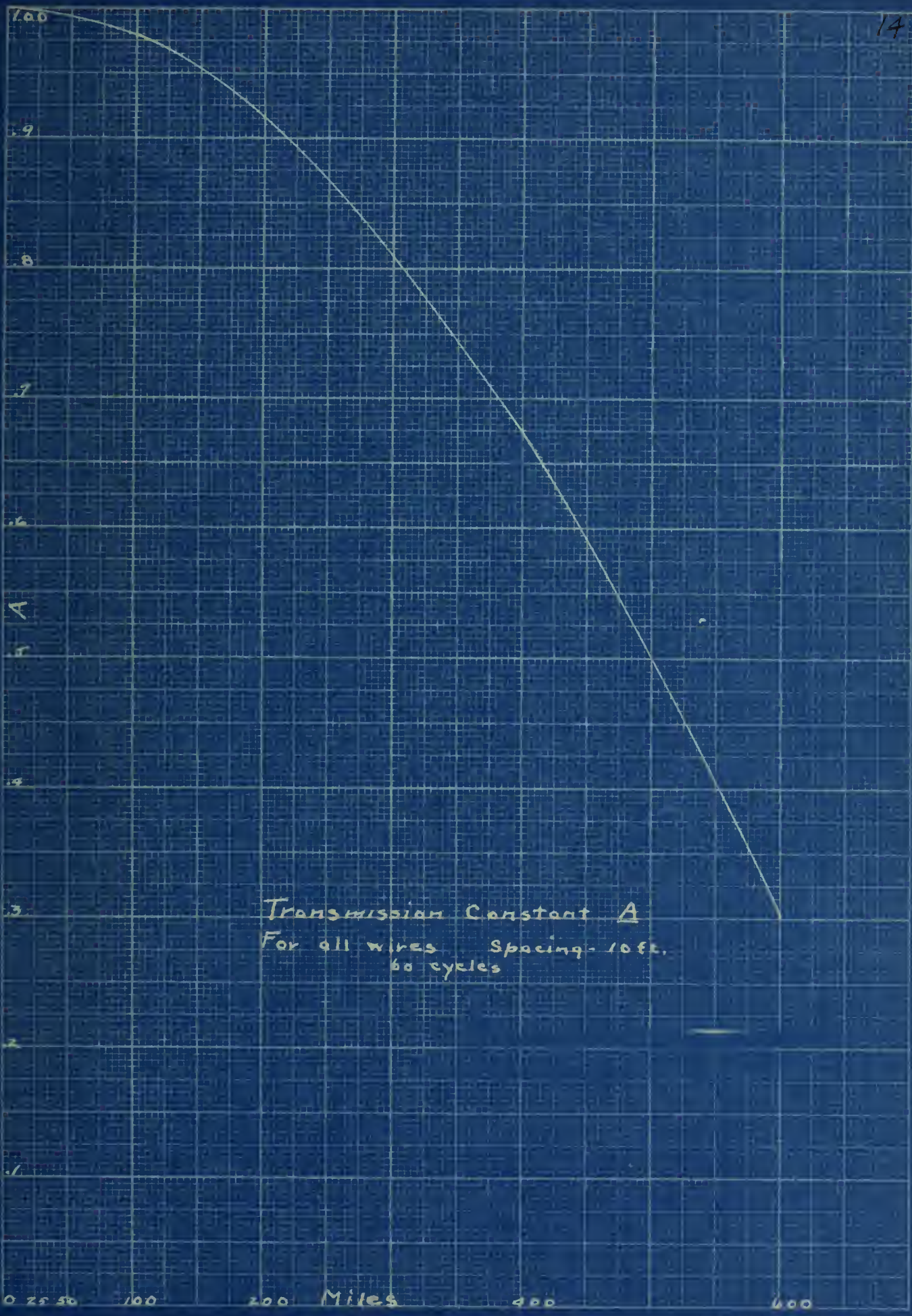
Distance in Miles	25	50	100	200	400	600
r_0	407.9	407.9	407.9	407.9	407.9	407.9
x_0	96.6	96.6	96.6	96.6	96.6	96.6
z_0	419.1	419.1	419.1	419.1	419.1	419.1
m_1	1.01	1.018	1.0276	1.0084	.8077	.3942
m_2	.0529	.1018	.22028	.45425	.9165	1.29
n_1	.986	.969	.93	.824	.54	.216
n_2	.0522	.10234	.1994	.371	.612	.708
$A = .5(m_1 + n_1)$	.99821	.99479	.98055	.91655	.67592	.3029
$B = .5(m_1 - n_1)$	.012	.0244	.0488	.0922	.13385	.0891
$C = .5(m_2 + n_2)$	.05255	.01207	.20984	.412625	.76425	.999
$D = .5(m_2 - n_2)$	.00035	.00473	.01044	.04162	.15225	.291

No. 000 WIRE. 10 FT. SPACING.
60 CYCLES.

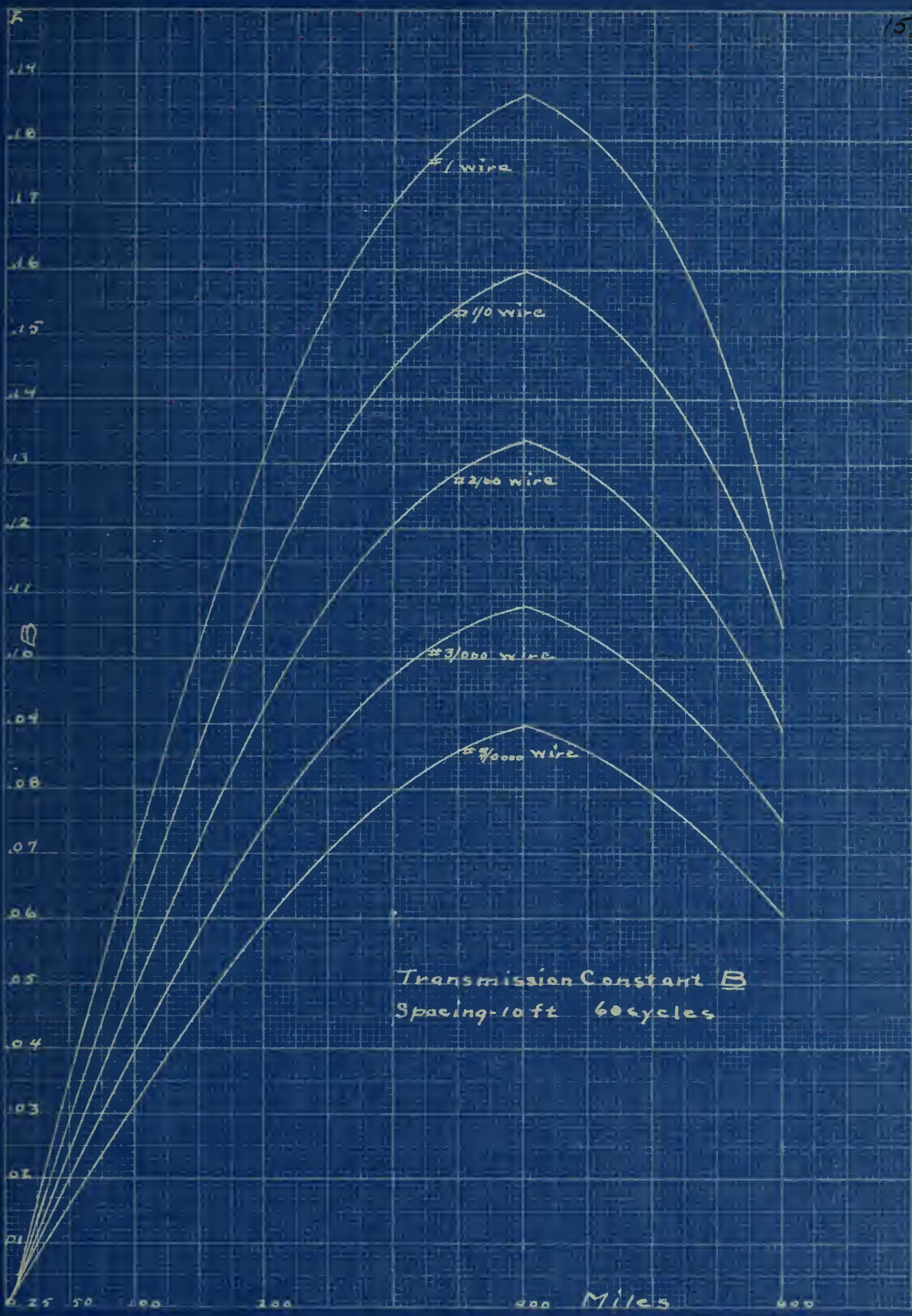
Distance in Miles	25	50	100	200	400	600
r_0	396.	396.	396.	396	396	396
x_0	76.	76.	76.	76.	76.	76.
z_0	403.3	403.3	403.3	403.3	403.3	403.3
m_1	1.0087	1.0147	1.0156	.991	.7875	.39487
m_2	.05	.106	.2283	.43947	.8706	1.21
n_1	.9886	.975	.938	.843	.571	.244
n_2	.0493	.1018	.211	.374	.633	.747
$A = 5(m_1 + n_1)$	.99821	.99479	.98055	.91655	.67592	.3029
$B = 5(m_2 - n_1)$	.01	.01985	.0387	.074	.10825	.07543
$C = 5(m_2 + n_2)$	.0496	.1039	.21968	.4067	.7518	.9785
$D = 5(m_2 - n_2)$	.00035	.0021	.00865	.0327	.1188	.2315

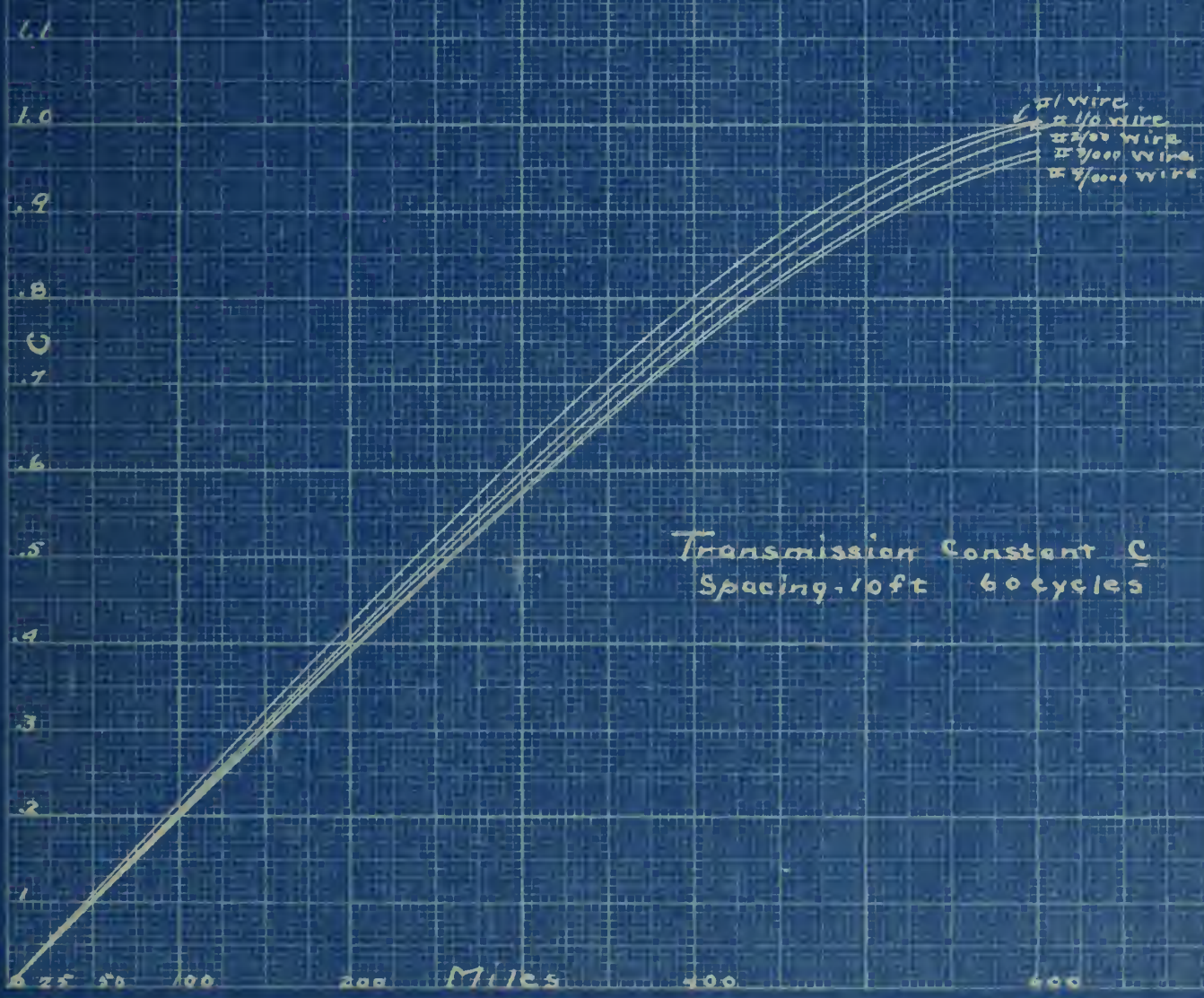
No. 0000 WIRE. 10 FT. SPACING.
60 CYCLES.

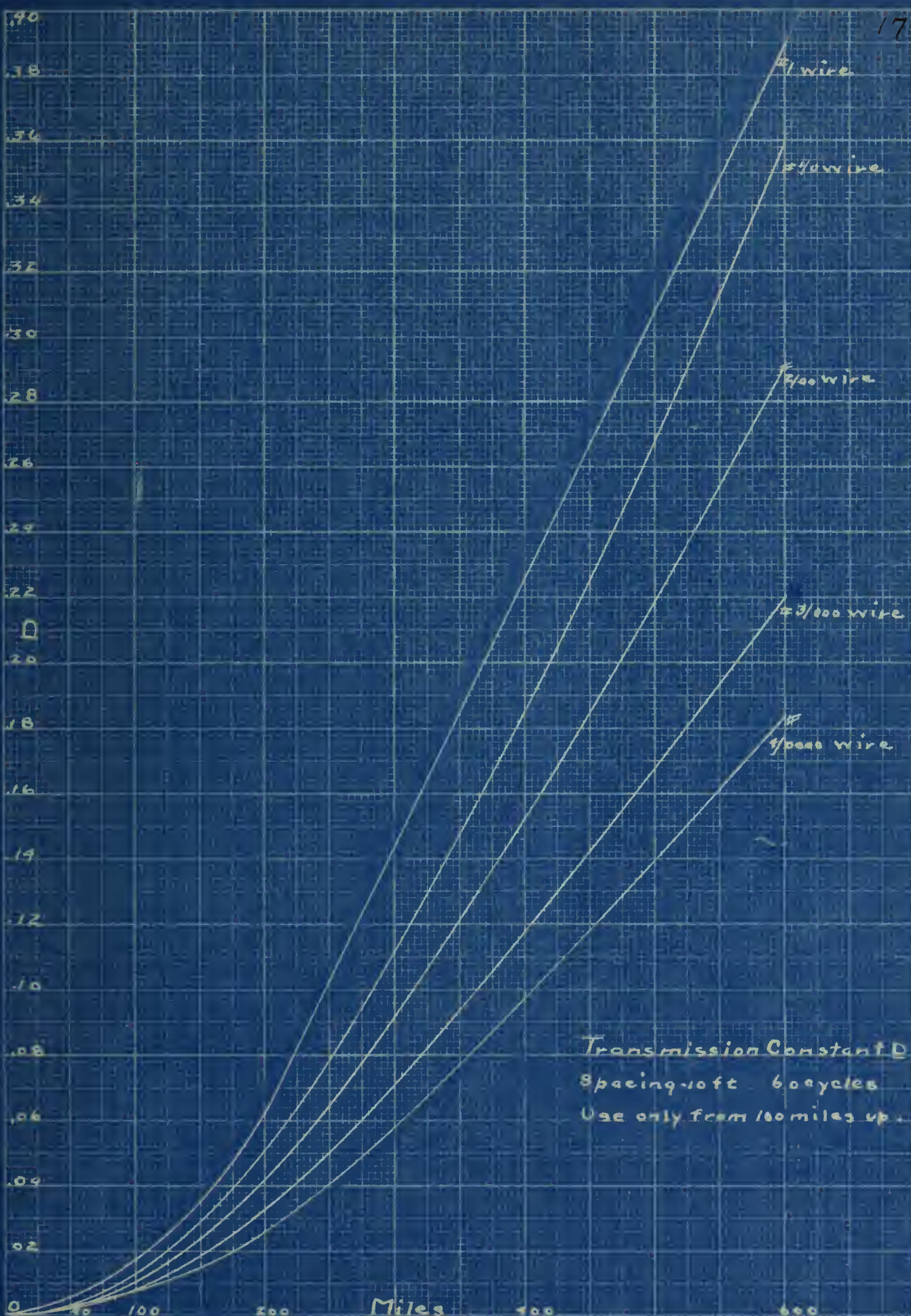
Distance in Miles	25	50	100	200	400	600
r_0	386.	386.	386.	386.	386.	386.
χ_0	59.3	59.3	59.3	59.3	59.3	59.3
z_0	390.5	390.5	390.5	390.5	390.5	390.5
m_1	1.0065	1.0094	1.01	.9744	.7704	.3765
m_2	.0545	.1062	.2142	.43216	.84269	1.15
n_1	.989	.98	.947	.856	.590	.2557
n_2	.0536	.103	.20	.38	.654	.784
$A = 5(m_1 + n_1)$	.99821	.99479	.98055	.91655	.67592	.3029
$B = 5(m_1 - n_1)$	.00875	.0147	.0315	.0592	.0902	.0604
$C = 5(m_2 + n_2)$	.05405	.1046	.2071	.40603	.7483	.967
$D = 5(m_2 - n_2)$	.00045	.0016	.0071	.02603	.0993	.1835



Transmission Constant A
For all wires Spacing - 10 ft.
60 cycles







No. 1 WIRE. 6 FT. SPACING. 60 CYCLES.

Distance in Miles	25	50	100	200	400	600
f_0	407.8	407.8	407.8	407.8	407.8	407.8
λ_0	146.6	146.6	146.6	146.6	146.6	146.6
Z_0	433.3	433.3	433.3	433.3	433.3	433.3
m_1	1.017	1.033	1.058	1.0584	.8840	.4426
m_2	.05561	.11408	.2340	.48917	1.04	1.535
n_1	.979	.955	.903	.778	.475	.1735
n_2	.0538	.1055	.2	.36	.557	.603
$A = .5(m_1 + n_1)$	.9981	.99435	.97897	.91659	.67805	.30855
$B = .5(m_1 - n_1)$	.019	.039	.0775	.1402	.2045	.125
$C = .5(m_2 + n_2)$	.05495	.1098	.2170	.42458	.7985	1.069
$D = .5(m_2 - n_2)$	.00116	.004294	.0170	.06458	.2415	.466

No. 0. WIRE. 6 FT. SPACING 60 CYCLES.

Distance in Miles	25	50	100	200	400	600
r_0	391.1	391.1	391.1	391.1	391.1	391.1
x_0	117.4	117.4	117.4	117.4	117.4	117.4
z_0	408.3	408.3	408.3	408.3	408.3	408.3
m_1	1.014	1.027	1.0413	1.0336	.849	.40299
m_2	.05465	.11096	.21799	.476	.982	1.4178
n_1	.982	.96	.9164	.800	.504	.185
n_2	.0529	.104	.2006	.368	.586	.653
$A = .5(m_1 + n_1)$	.9981	.99435	.97897	.91659	.67805	.30709
$B = .5(m_1 - n_1)$	.016	.0335	.06245	.1168	.1725	.10899
$C = .5(m_2 + n_2)$	.05379	.10748	.20929	.422	.7840	1.0354
$D = .5(m_2 - n_2)$	.000875	.00348	.00869	.054	.1980	.3824

No.00. WIRE. 6 FT. SPACING. 60 CYCLES.

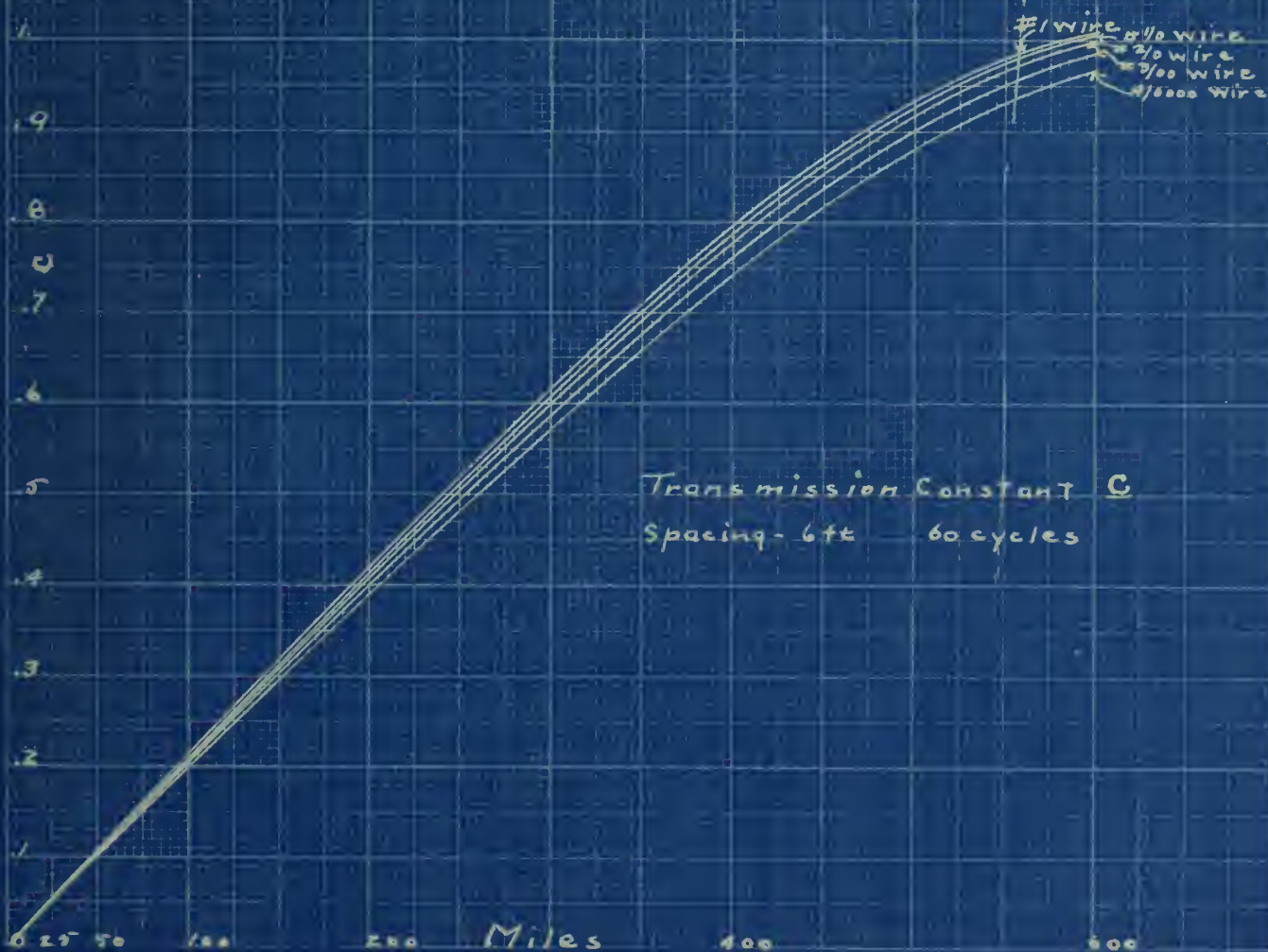
Distance in Miles	25	50	100	200	400	600
I_0	380.0	380.0	380.0	380.8	380.0	380.0
X_0	94.6	94.6	94.6	94.6	94.6	94.6
Z_0	391.5	391.5	391.5	391.5	391.5	391.5
M_1	1.0115	1.02	1.032	1.0126	.8156	.3984
M_2	.053689	.1084	.2223	.45836	.93855	1.315
N_1	.984	.97	.926	.827	.534	.204
N_2	.0524	1.03	.20	.3715	.607	.698
$A = .5(M_1 + N_1)$	.9981	.99435	.97897	.91659	.67805	.30709
$B = .5(M_1 - N_1)$	.01375	.025	.053	.0963	.1408	.09723
$C = .5(M_2 + N_2)$	.05304	.1057	.2115	.4149	.77277	1.0055
$D = .5(M_2 - N_2)$	.000644	.0027	.01115	.04348	.165375	.3095

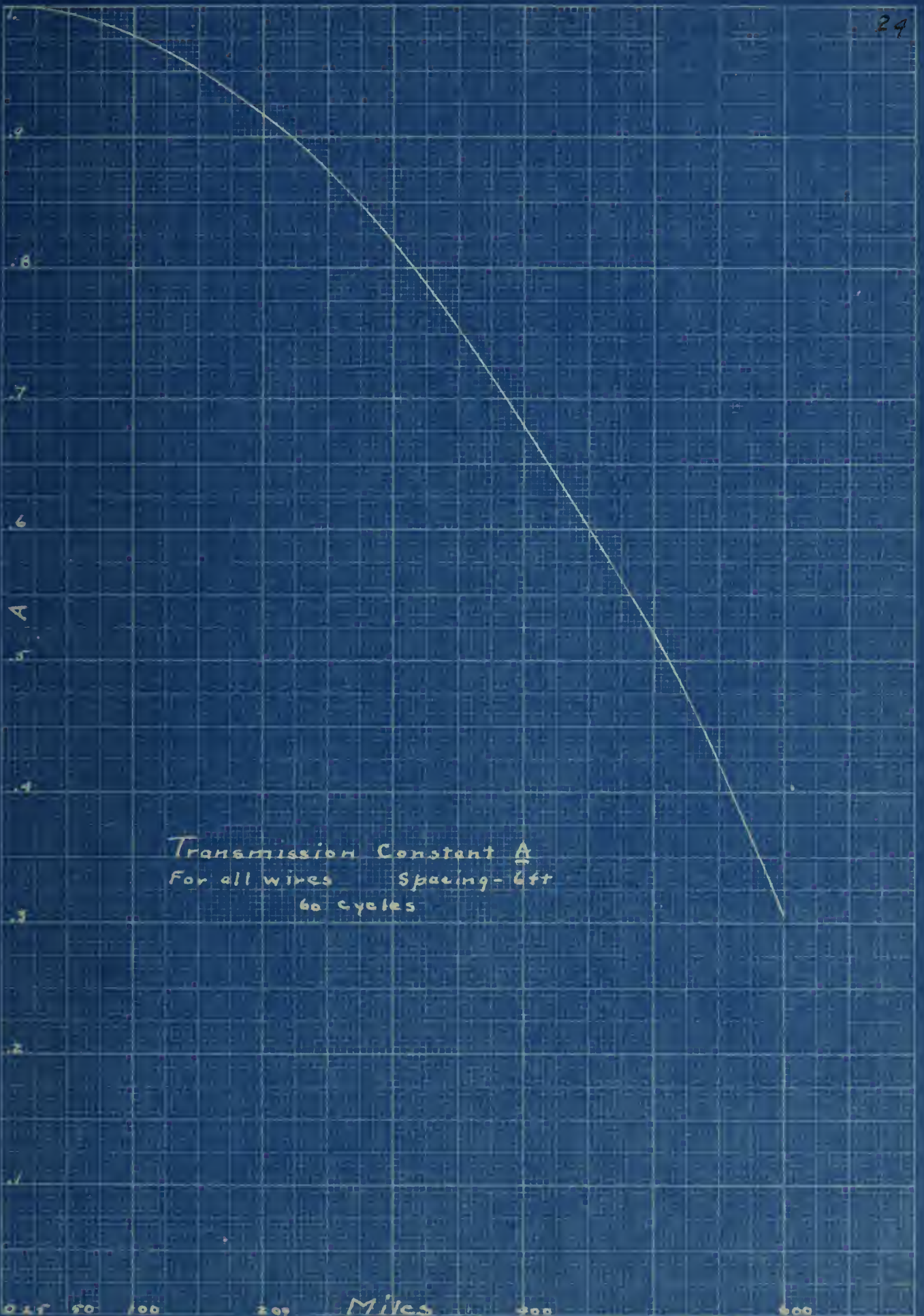
No.000 WIRE. 6 FT. SPACING. 600 CYCLES.

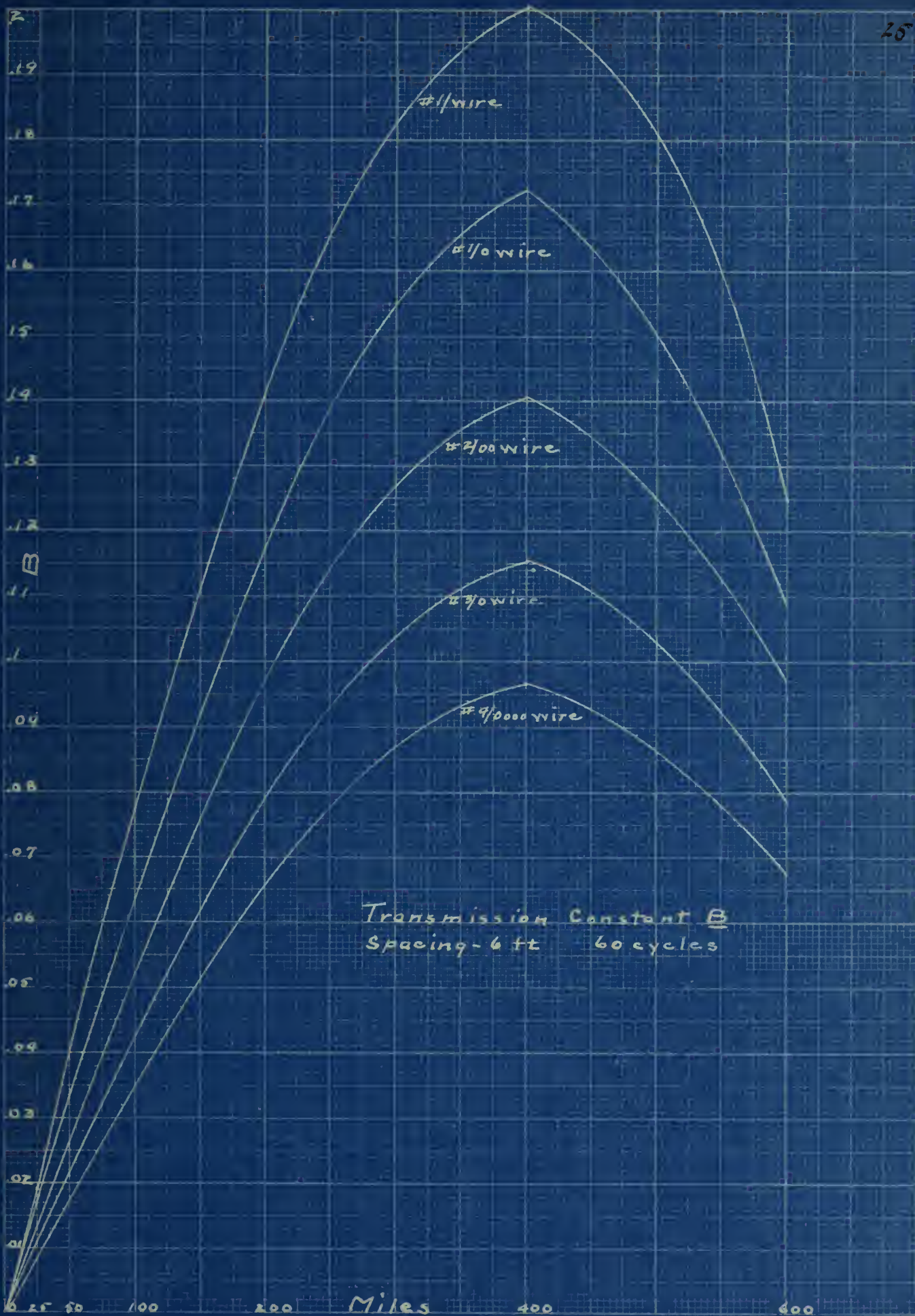
Distance in Miles	25	50	100	200	400	600
r_0	367.8	367.8	367.8	367.8	367.8	367.8
x_0	75.6	75.6	75.6	75.6	75.6	75.6
z_0	375.3	375.3	375.3	375.3	375.3	375.3
m_1	1.0096	1.0163	1.0211	.99476	.7938	.39255
m_2	.0529	.1071	.21736	.44537	.8837	1.235
n_1	.9877	.973	.936	.836	.563	.283
n_2	.05177	.1023	.199	.375	.627	.733
$A = .5(m_1 + n_1)$	.9981	.99455	.97897	.91659	.67805	.30709
$B = .5(m_1 - n_1)$	.0109	.02165	.0425	.07938	.1154	.07977
$C = .5(m_2 + n_2)$	.05231	.1047	.20817	.410185	.75535	.984
$D = .5(m_2 - n_2)$	.00059	.00241	.009175	.03518	.12835	.251

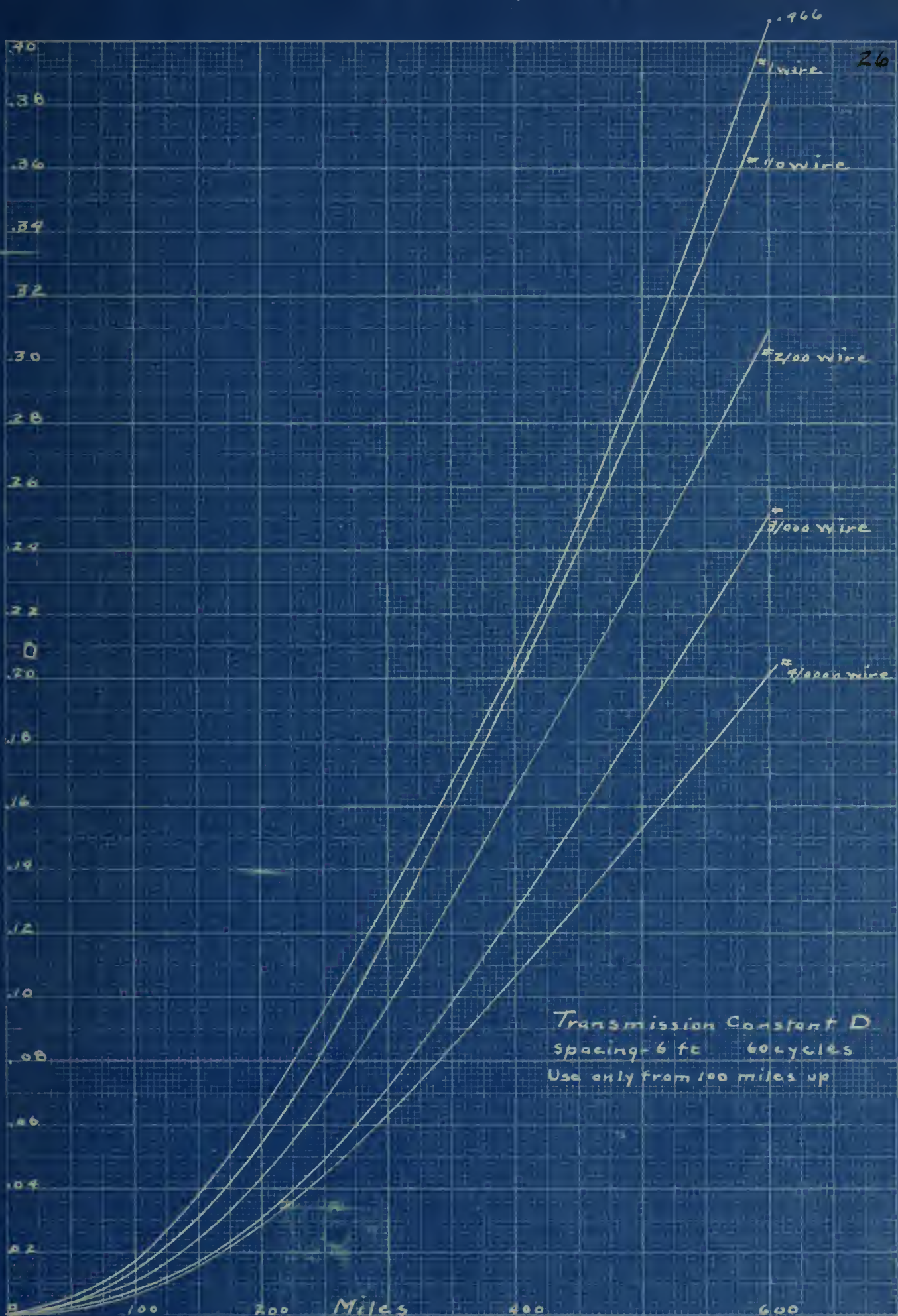
No. 0000 WIRE. 6 FT. SPACING. 60 CYCLES.

Distance in Miles	25	50	100	200	400	600
f_0	360.67	360.67	360.67	360.67	360.67	360.67
λ_0	60.91	60.91	60.91	60.91	60.91	60.91
Σ_0	365.62	365.62	365.62	365.62	365.62	365.62
m_1	1.007	1.00654	1.014	.98136	78148	.39
m_2	.0522	.105	.21215	.4338	.84639	1.17
n_1	.99	.983	.945	.85	.587	.2544
n_2	.0512	.102	.1975	.376	.635	.766
$A = .5(m_1 + n_1)$	.9981	.99435	.97897	.91659	.68424	.30709
$B = .5(m_1 - n_1)$	.0085	.01675	.0345	.06568	.09674	.0678
$C = .5(m_2 + n_2)$	.0517	.1035	.20482	.4049	.74064	.968
$D = .5(m_2 - n_2)$	.0005	.0015	.00732	.0289	.10569	.202













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